under axial pressure contains about 50% membrane stress, he concludes that $\sigma^c = \frac{1}{2}\sigma^c_{(\text{classical})}$ is a lower bound estimate and rules out the influence of boundary conditions. However, I have calculated the membrane contribution for the critical load of the free edge cylindrical shell following Croll and found that according to his theory a lower bound must be $\sigma^c = \frac{1}{4}\sigma^c_{(\text{classical})}$ and not $\sigma^c = \frac{1}{2}\sigma^c_{(\text{classical})}$. It is also interesting to note that imperfection sensitivity in some axially inextensional structures can exist due to nonlinear compatibility conditions and regardless of the amount of membrane strain energy as pointed out in detail in Refs. 4 and 5.

Having said that, we must, of course, admit that the works of the second and third groups have, nevertheless, illuminated many dark corners and clarified for us many ambiguous points.

References

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Comment on "Inviscid Hypersonic Flow around a Semicone Body"

P.L. Roe*
Royal Aircraft Establishment, Bedford, U.K.

KIMURA and Tsutahara propose in Ref. 1 a solution for the inviscid hypersonic flow past a flat topped semicone body. Having established by an unsteady analogy that the cross-flow velocity components obey the Laplace equation they proceed to solve this under boundary conditions which include an assumption of vanishing velocities at infinite radius. They then combine the known density ratio across shock waves at infinite Mach number with the continuity equation to obtain the shock shape from the calculated velocities. Since this interposes a discontinuity between the body and the boundary condition at infinity, one is inclined to suspect the relevance of that condition. In fact, that boundary condition can only be satisfied because a genuinely relevant condition has been omitted. Consider a conical shockwave $r/x = r_s(\theta)$, normal to which a vector in (x,r,θ) coordinates may be written as $n = (r_s, -1, r_s^{-1}\mathrm{d}r_s/\mathrm{d}\theta)$. The equation for continuity of mass may be written $\rho_1(q_1 \cdot n) = \rho_2(q_2 \cdot n)$ or

$$v_\theta \frac{\mathrm{d}r_s}{\mathrm{d}\theta} + r_s^2\left[v_x - \frac{\gamma-1}{\gamma+1}U\right] - v_r r_s = 0 \tag{1}$$

The equation for continuity of momentum parallel to the shock is

$$q_1 \times n = q_2 \times n$$

which yields two independent equations

$$U - v_x - v_r r_s = 0 \tag{2}$$

$$v_r \frac{\mathrm{d}r_s}{\mathrm{d}\theta} + v_\theta r_s = 0 \tag{3}$$

Equation (2) may be interpreted as $v_x = U + O(r^2/x^2)$, which is intuitive anyway, and combined with Eq. (1) to yield

$$v_\theta \frac{\mathrm{d}r_s}{\mathrm{d}\theta} + \left[1 - \frac{\gamma-1}{\gamma+1}\right] r_s^2 - v_r r_s = 0 \tag{4}$$

which is Eq. (10) of Ref. 1, and is employed there to find the shock shape. Equation (3), however, is not considered at all, although it has quite equal status with Eq. (4) as an identity which must be satisfied on the shock. The condition for (3) and (4) to be compatible is

$$v_r^2 + v_\theta^2 = 2v_r r_s/(\gamma+1) \tag{5}$$

It is this boundary condition, to be applied on the initially unknown shock-wave, which should replace the spurious boundary condition at infinity. It should be noted that Eq. (3), and hence Eq. (5), is automatically satisfied in axisymmetric flow ($v_\theta = 0$), or two-dimensional flow ($v_\theta = v_r \tan\theta$), and it would seem safe to neglect it in situations close to one of these. However, inspection of the results of Ref. 1 indicated a large region, roughly $\pi/4 < |\theta| < \pi/2$, where neither assumption is satisfied. Thus, although the flow in $|\theta| < \pi/4$ may be self-consistent, it cannot be regarded as caused by removing the cone top, since the regions in which cause and effect operate are separated by a region in which the flow model breaks down.

Reference

[1]Kimura, T. and Tsutahara, M., "Inviscid Hypersonic Flow around a Semicone Body," *AIAA Journal*, Vol. 13, Oct. 1975, pp. 1349-1353.

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Reply by Authors to P. L. Roe

Takeyoshi Kimura* and Michihisa Tsutahara†
Kobe University, Nada, Kobe, Japan

IT is natural that the condition for Eq. (3) in Roe's comment must be considered. However such momentum relations are sometimes neglected in constant density solutions.[1] For a complete solution, the relations between the flows of the two sides, before and behind the shock wave, such as the conservation equation for energy and the relations of entropy change, must be satisfied, as well as Eq. (3). Since the strength of the shock wave cannot be the same everywhere, they are contradictory to our assumption that the internal energy and entropy are uniform within the shock layer. Therefore we do not think that the previous assumption leads a satisfactory solution for the whole flowfield.

Mr. Roe says that it is Eq. (5) in his Comment, to be applied on the initially unknown shock wave, which should replace our spurious boundary condition at infinity. However, it is difficult to think that this will improve our analysis, because the contradictions in the region where θ is large are essentially

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*Professor, Dept. of Mechanical Engineering. Member AIAA.

†Research Assistant, Dept. of Mechanical Engineering. Member AIAA.